Momentum and Collisions I Cheat Sheet

Conservation of Linear Momentum

The momentum of a moving object is given by the product of its mass (m) and velocity (v). The units for momentum are kgms^{-1} or N, which can be used interchangeably.

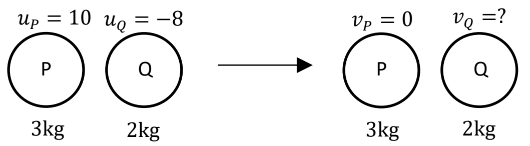
$$p = mv$$

Newton's third law states that the force exerted by object A on object B is the same as the force exerted by object B on object A but in the opposite direction. Thus when 2 objects collide, the increase in momentum of one object is equal to the decrease in momentum of the other. The law of conservation of momentum states that in an isolated system without external influence, the total momentum before the collision is the same as after the collision. Therefore, for objects A and B with masses m_A and m_B, initial velocities u_A and u_B, and final velocities v_A and v_B:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

1D Motion

Example 1: A particle P has a mass of 3kg and is travelling along a smooth horizontal surface with velocity 10ms^{-1}. It collides with particle Q which has a mass of 2kg and is travelling in the opposite direction with a velocity of -8ms^{-1}. **a.)** Find the momentum of particle P before the collision. **b.)** Given that particle P comes to a complete stop after the collision, show that the direction of travelling of Q is reversed and find its final velocity. **c.)** Given that the two particles coalesce after the collision, find their velocity.

a.) Find the momentum of P using $p = mv$.	$p = (3\text{kg})(10\text{ms}^{-1})$ $= 30\text{kgms}^{-1}$
b.) Draw a diagram to visualise the question. Take the direction towards the right at the positive direction.	
Using $m_P u_P + m_Q u_Q = m_P v_P + m_Q v_Q$, find the velocity of Q after collision.	$(3)(10) + (2)(-8) = (3)(0) + (2)v_Q$ $30 - 16 = 2v_Q$ $v_Q = \frac{14}{2}$ $= 7\text{ms}^{-1}$ u_Q is in the positive direction and the change in sign in v_Q indicates a reverse in the direction of travelling.
c.) When 2 particles coalesce, they move together as one object so their masses should be added together.	$(3)(10) + (2)(-8) = (3+2)v_{P+Q}$ $14 = 5v_{P+Q}$ $v_{P+Q} = 2.8\text{ms}^{-1}$

2D Motion

Momentum and velocity are vectors which can be written in terms of i and j. The momentum in direction i and direction j will be conserved independently.

Example 2: A particle P with a of mass of 2kg and is travelling at $(6i + 4j)\text{ms}^{-1}$. It collides with the particle Q which has a mass of 1.5kg and is at rest. Given that particle P is moving at $(2i - j)\text{ms}^{-1}$ after the collision, find the speed of particle Q.

Find the velocity of Q using the law of conservation of momentum, writing the velocities in terms of i and j.	$2(6i + 4j) + 1.5(0i + 0j) = 2(2i - j) + 1.5(v_Q)$ $12i + 8j = 4i - 2j + 1.5v_Q$ $8i + 10j = 1.5v_Q$ $v_Q = \frac{16}{3}i + \frac{20}{3}j\text{ms}^{-1}$
Find the speed of particle Q by finding the magnitude of its velocity vector.	$\sqrt{\left(\frac{16}{3}\right)^2 + \left(\frac{20}{3}\right)^2} = \sqrt{\frac{256}{9} + \frac{400}{9}}$ $= \sqrt{\frac{656}{9}}$ $\approx 8.54\text{ms}^{-1}$

Restitution and Newton's Experimental Law

The coefficient of restitution is denoted by the constant e, with $0 \le e \le 1$.

According to Newton's experimental law,

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

The value of e is dependent on the material of the object and determines the amount of kinetic energy lost. $e = 1$ is known as a perfectly elastic collision and total kinetic energy is not lost. When $e = 0$, there is a loss in kinetic energy and the two objects coalesce and move together as one particle. This is known as a perfectly inelastic collision.

Direct Collisions

Example 3: 2 particles, P and Q, are travelling towards each other with velocities 5ms^{-1} and $-2ms^{-1}$ respectively. Given that they move away from each other after the collision at -2.5ms^{-1} and 1.5ms^{-1} respectively, find the coefficient of restitution between the two particles.

Write down the velocities of each particle before and after the collision, keeping in mind to indicate their directions using signs.	$u_P = 5, u_Q = -2$ $v_P = -2.5, v_Q = 1.5$
Use Newton's experimental law to find e.	$e = \frac{1.5 - (-2.5)}{5 - (-2)}$ $= \frac{4}{7}$

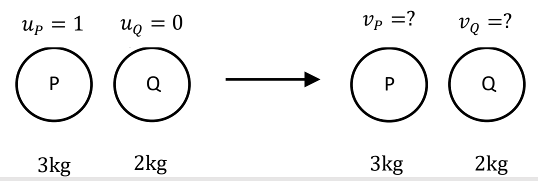
Collision Between a Particle and a Fixed Object

When a particle collides with a fixed object (such as a wall) at a right angle to its movement, the law of conservation of momentum cannot be used. In this case, Newton's experimental law is used to find the final velocity of the particle.

$$e = \frac{0 - v_1}{u_1 - 0}$$

$$v_1 = -eu_1$$

Example 4: Particles P and Q lie on a smooth surface in a straight line perpendicular to a fixed wall. Particle P has a mass of 3kg and is projected towards particle Q at a velocity of 1ms^{-1}. Particle Q has a mass of 2kg and is initially at rest. After the first collision, particle Q collides with the wall at a right angle. **a.)** Given that the coefficient of restitution between the two particles is 0.4, find the speed and direction of travelling for particles P and Q after the first collision. **b.)** Find the velocity of particle Q after the second collision when the coefficient of restitution between particle Q and the wall is 0.6. **c.)** State whether there will be a third collision, giving a reason for your answer.

a.) Draw a diagram for the first collision.	
Form an equation for the velocity of particles P and Q after the first collision using the law of conservation of momentum.	$3(1) + 2(0) = 3v_P + 2v_Q$ $3 = 3v_P + 2v_Q$
Form a second equation for the velocity of particles P and Q after the first collision using Newton's experimental law.	$0.4 = \frac{v_Q - v_P}{1 - 0}$ $0.4 = v_Q - v_P \Rightarrow 0.8 = 2v_Q - 2v_P$
Solve for v_P and v_Q by adding the two equations together.	$3v_P + 2v_Q = 3$ $2v_Q - 2v_P = 0.8$ $5v_P = 2.2 \Rightarrow v_P = 0.44\text{ms}^{-1}$ $v_Q = 0.4 + 0.44 = 0.84\text{ms}^{-1}$
b.) Use Newton's experimental law to find v.	$v = -eu = -(0.6)(0.84)$ $= -0.504\text{ms}^{-1}$
c.) Look at the direction of travel of the particles and their speed to determine whether they will collide.	Particle P is travelling in the positive direction (towards particle Q and the wall) while particle Q is travelling in the negative direction. The particles are moving towards each other so they will collide again.